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DYNAMIC ANALYSIS OF A MULTI-LEGGED LUNAR LANDING VEHICLE TO DETERMINE STRUCTURAL LOADS DURING TOUCHDOWN

by John Admire and Alden Mackey

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DEFINITION OF SYMBOLS

SYMBOL	DEFINITION
a	One-half the chord between leg attachment points 3 and 4 on radius R_2 (in.)
A_{ijk}	Component of ℓ_{ijk} in xy plane (in.)
F_{ij5}	Force in the struts joining points j and 5 for the i^{th} leg (lb)
$\overline{F}_{i6x}$	Force in the x direction applied to the footpad by the i^{th} leg (lb)
$\overline{F}_{i6y}$	Force in the y direction applied to the footpad by the i^{th} leg (lb)
$\overline{F}_{i6z}$	Force in the z direction applied to the footpad by the i^{th} leg (lb)
$\overline{F}^F_{i6x}$	Friction force between surface and footpad acting in the x direction (lb)
$\overline{F}^F_{i6y}$	Friction force between surface and footpad acting in the y direction (lb)
$\overline{F}^N_{i6z}$	Reaction of the surface to load imposed by vehicle and is equal in magnitude to F_{i6z} (lb)
g	Local gravitational acceleration (in./sec^2)
I	Mass moment of inertia about the y' axis (lb-in.-sec^2)
$K_{\delta_{ij5}}$	Spring constant of the strut between the j and 5 points on the i^{th} leg (lb/in.)
$K_{c_{ij5}}$	Effective compression spring constant of the strut between the j and 5 points when the onset rate of the honeycomb and the spring in the strut are combined in series (lb/in.)
$\overline{K}_{c_{ij5}}$	Effective tension spring constant of the strut between the j and 5 points when the onset rate of the honeycomb and the spring in the strut are combined in series (lb/in.)
ℓ_{ijk}	Length of the strut between points j and k on the i^{th} leg (in.)

DEFINITION OF SYMBOLS (Cont'd)

SYMBOL	DEFINITION
m	Mass of the complete vehicle $\left(\frac{lb\text{-}sec^2}{in.}\right)$
m_p	Mass of the footpads $\left(\frac{lb\text{-}sec^2}{in.}\right)$
$\overline{m}$	Number of legs which must be analyzed for each time increment
n	Total number of legs on the vehicle
P_{ij}	Coordinates of the points on the vehicle in surface-fixed coordinate system
P'_{ij}	Coordinates of the points on the vehicle in body-centered coordinate system
R_{ij5}	Length of strut between joints j and 5 of the i^{th} legs if all external forces are removed after an initial deformation (in.)
R_1	Radial distance from longitudinal axis to point 2 on the vehicle (in.)
R_2	Radial distance from longitudinal axis to either point 3 or 4 on the vehicle (in.)
R_3	Radial distance from the longitudinal axis to point 5 on the vehicle (in.)
R_4	Radial distance from the longitudinal axis to point 6 on the vehicle (in.)
Δt	Time increment (sec)
T	Time (sec)
V_{ij5}	Maximum compression force which can be carried by the crushable material in a strut (lb)

DEFINITION OF SYMBOLS (Cont'd)

SYMBOL	DEFINITION
$\overline{V}_{ij5}$	Maximum tension force which may be carried by the crushable material in a strut (lb)
V_V	Initial vertical velocity of the vehicle (in./sec)
V_H	Initial horizontal velocity of the vehicle (in./sec)
x_{ij}, y_{ij}, z_{ij}	Coordinates of vehicle points in the surface-fixed coordinate system (in.)
x'_{ij}, y'_{ij}, z'_{ij}	Coordinates of vehicle points in the body-centered coordinate system (in.)
$\dot{x}_{i1}$	Velocity parallel to x axis of the vehicle's center of gravity (in./sec)
$\dot{z}_{i1}$	Velocity parallel to z axis of the vehicle's center of gravity (in./sec)
$\ddot{x}_{i1}$	Acceleration parallel to x axis of vehicle's center of gravity (in./sec)
$\ddot{z}_{i1}$	Acceleration parallel to z axis of vehicle's center of gravity (in./sec)
$\dot{x}_{i6}$	Velocity of the i^{th} footpad parallel to the x axis (in./sec)
$\dot{y}_{i6}$	Velocity of the i^{th} footpad parallel to the y axis (in./sec)
$\ddot{x}_{i6}$	Acceleration of the i^{th} footpad parallel to the x axis (in./sec^2)

SYMBOL	DEFINITION
$\ddot{y}_{i6}$	Acceleration of the i^{th} footpad parallel to the y axis (in./sec^2)
x_{Hij}	Coordinates of the vehicle relative to a horizontal axis (in.)
γ_{i65}	Angle between a line from point 5 to 6 and its projection in the x-y plane (rad)
γ_{i62}	Angle between a line from points 2 and 6 and its projection in the x-y plane (rad)
Δ_1	Angle between any two joining legs (Fig. 2) (rad)
Δ_2	Angle between two planes, the first defined by the longitudinal axis and P_{i2} and the second defined by the longitudinal axis and P_{i3} (rad)
Δ_3	The angle between two planes, the first defined by the longitudinal axis and P_{i2} and the second the x-z plane (rad)
Δ_{ij5}	The deformation in compression that ℓ_{ij5} has undergone when the maximum honeycomb crush force in the strut is attained (in.)
$\overline{\Delta}_{ij5}$	Same as Δ_{ij5} but for tension loads (in.)
Δ'_{ij5}	The elastic deformation of ℓ_{ij5} in compression (in.)
$\overline{\Delta}'_{ij5}$	Same as Δ'_{ij5} but for tension (in.)
β	Slope of the lunar surface at the touchdown point (rad)
δ_{ij5}	Onset distance of honeycomb material in compression (in.)
$\overline{\delta}_{ij5}$	Same as δ_{ij5} but in tension (in.)
μ	Coefficient of friction between the lunar surface and the footpads of the vehicle

DEFINITION OF SYMBOLS (Concluded)

SYMBOL	DEFINITION
θ	Angle between the longitudinal axis of the vehicle and a vertical line measured in the x-z plane (rad)
$\dot{\theta}$	Time rate of change of θ (rad/sec)
ϕ	Angle between the x and x' axis (rad)
$\dot{\phi}$	Time rate of change of ϕ (rad/sec)
$\zeta_{i52,\ i56}$	The angle between the lines from P_{i2} to P_{i5} and P_{i5} to P_{i6} (rad)

DYNAMIC ANALYSIS OF A MULTI-LEGGED LUNAR LANDING VEHICLE TO DETERMINE STRUCTURAL LOADS DURING TOUCHDOWN

SUMMARY

This report presents a method of determining the structural loads of a multi-legged lunar landing vehicle during touchdown. The stability of the vehicle is obtained as a fundamental aspect of the problem. The motion of the vehicle center of gravity is restricted to two dimensions, but the feet of the vehicle, while on the lunar surface, may move in any direction. The feet are connected to the body of the vehicle by deformable legs. These legs consist of an inverted tripod and a member from the tripod vertex to the foot.

The ability of the feet to move in any direction while on the lunar surface is a significant improvement over a strictly two-dimensional analysis. A more accurate evaluation of the loads in the struts can be made and, therefore, the actual dynamics of the vehicle are better defined. The capability of this procedure to analyze the touchdown dynamics of a vehicle with any number of legs is also a significant advancement over the two-dimensional analysis.

SECTION I. INTRODUCTION

A dynamic analysis of the touchdown of a lunar vehicle is important for two reasons: to determine dynamic loads and to determine stability (loads or stability could cause a failure of the lunar mission). The mathematical model used in this analysis is sufficiently sophisticated to determine loads and stability within the accuracy of the parameters used, such as coefficient of friction, slope, and texture of the lunar surface.

This mathematical model has the capability of analyzing any number of legs, each of which is made up of four members. Three of the members form an inverted tripod, and the fourth member connects the vertex of the tripod to the footpad. Each member of the tripod may deform in either tension or compression, this deformation being elastic or plastic, or both.

The center of gravity of the vehicle is restricted to motion in a plane, but the motion of the footpads, while on the lunar surface, may be in any direction, subject only to the restraints imposed on them by the lunar surface. These restraints, such as friction between lunar surface and footpads and slope of lunar surface, may assume any value.

As more is learned about the lunar surface we may find that its bearing strength is much lower than is now believed. Should this be true, a leg type landing vehicle would not be feasible, and lunar landing vehicles would require more surface in contact with the lunar surface. If a torus, or other similar configuration were used, it could be analyzed by the method presented herein by using a large number of legs to approximate continuous contact with the lunar surface.

SECTION II. THEORETICAL DYNAMIC ANALYSIS

A. METHOD OF ANALYSIS

A closed-form solution of the dynamic equations of motion would require solving $3 + 2n$, where n = number of legs, simultaneous nonlinear second-order differential equations. The complexity of these equations requires that a numerical method be used to obtain the solution.

To reduce the equations to ordinary linear differential equations, the following assumption is made: all forces acting on the system are considered constant during short times of integration of the differential equations. Based on this assumption, the equations are easily solved.

An outline of the steps necessary to develop the time history of the system is best described by a simple block diagram (Fig. 1). The details of the computation for this method will now be developed.

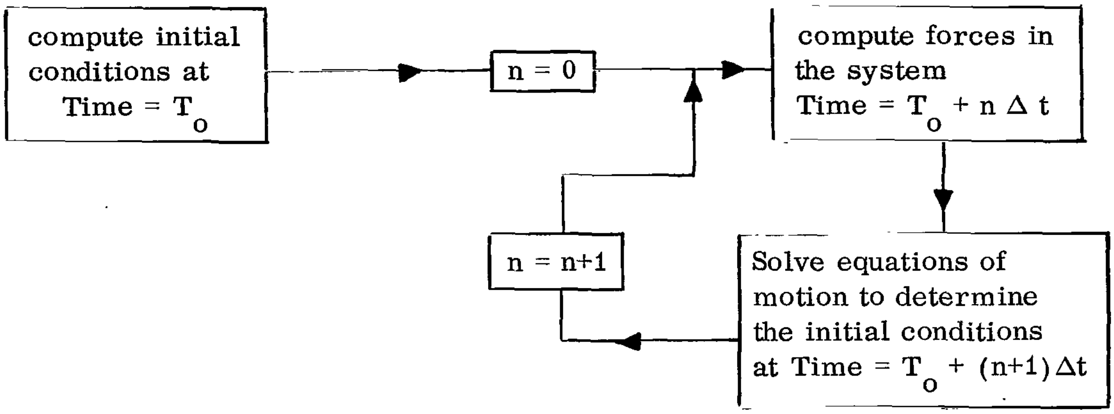

FIGURE 1. SYSTEM TIME HISTORY, BLOCK DIAGRAM

B. GEOMETRY OF DYNAMIC MODEL

The complete geometry of the vehicle is described by the positions of the several reference points on the model. These points are the center of gravity and five points on each of the legs. The points P_{ij} are shown in Figure 2 (i refers to the leg, and j refers to the specific point on that leg). Note here that P_{i1} always refers to the center of gravity of the vehicle.

It was convenient to use two coordinate systems in this analysis (Fig. 3). The equations of motion of the vehicle are written in terms of a coordinate system which is fixed to the lunar surface. Position and force vectors are determined using another coordinate system which is fixed to the vehicle.

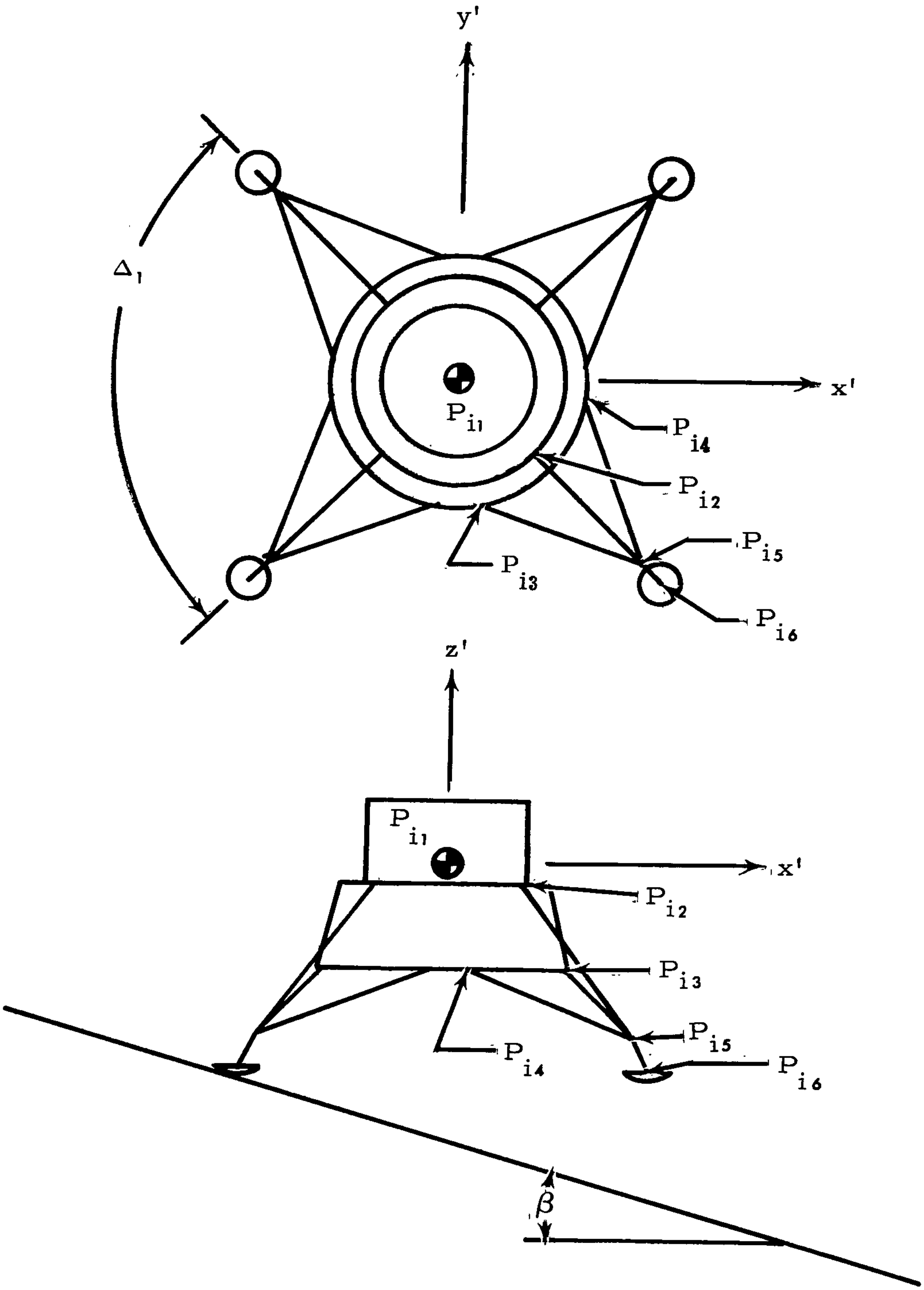

FIGURE 2. VEHICLE GEOMETRY AND THE BODY-FIXED COORDINATE SYSTEM

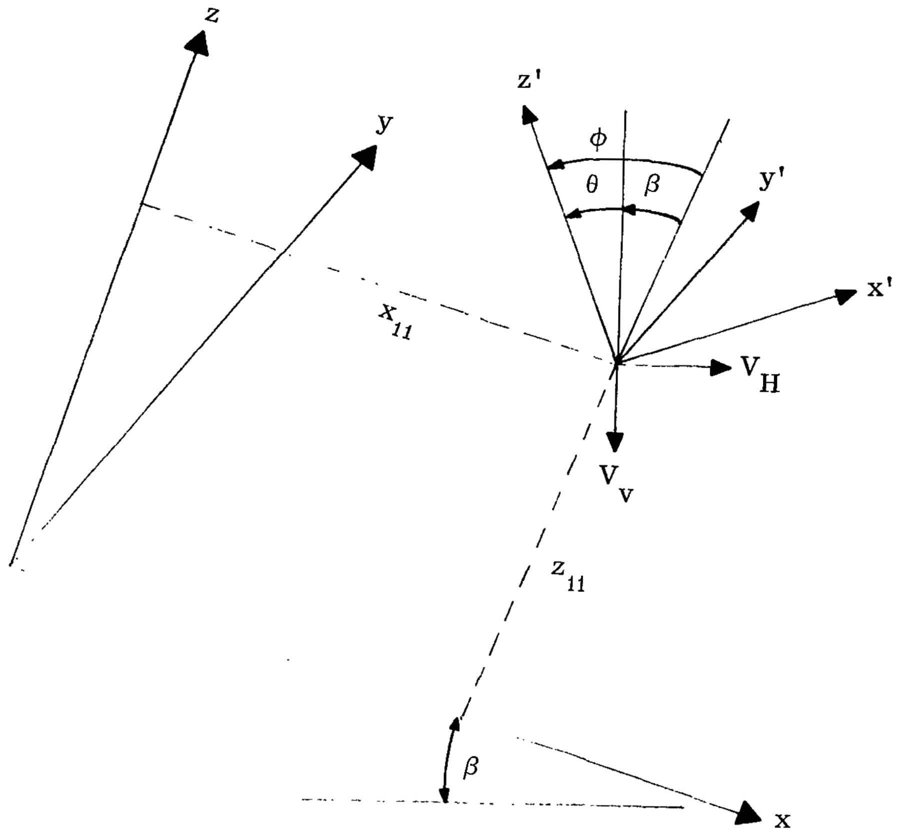

FIGURE 3. BODY-FIXED AND SURFACE-FIXED COORDINATE SYSTEMS

The lunar-fixed coordinate system is oriented so the x-y plane lies on the lunar surface with the x-axis in the direction of maximum slope and the z-axis is normal to the lunar surface. The body-fixed coordinate system has its origin at the center of gravity of the vehicle and the z'-axis along the longitudinal axis of the vehicle. The x'-z' plane lies in the x-z plane, and the y'-axis is parallel to the y-axis. The transformation from one coordinate system to another can be written as follows:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x_o \\ y_o \\ z_o \end{bmatrix} + \begin{bmatrix} \cos\phi & 0 & \cos(\pi/2 + \phi) \\ 0 & 1 & 0 \\ \cos(\pi/2 - \phi) & 0 & \cos\phi \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} \tag{1}$$

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \cos\phi & 0 & \cos(\pi/2 - \phi) \\ 0 & 1 & 0 \\ \cos(\pi/2 + \phi) & 0 & \cos\phi \end{bmatrix} \begin{bmatrix} x - x_o \\ y \\ z - z_o \end{bmatrix} \tag{2}$$

For specific points on the vehicle, equations 1 and 2 take the following form:

$$P_{ij} = P_{i1} + AP'_{ij} \tag{3}$$

$$P'_{ij} = B\,(P_{ij} - P_{i1})$$

$$P_{ij} = \begin{bmatrix} x_{ij} \\ y_{ij} \\ z_{ij} \end{bmatrix} \qquad P = \begin{bmatrix} x_{i1} \\ y_{i1} \\ z_{i1} \end{bmatrix} \qquad P'_{ij} = \begin{bmatrix} x_{ij} \\ y_{ij} \\ z_{ij} \end{bmatrix}$$

$$A = \begin{bmatrix} \cos\phi & 0 & \cos(\pi/2 + \phi) \\ 0 & 1 & 0 \\ \cos(\pi/2 - \phi) & 0 & \cos\phi \end{bmatrix}$$

$$B = \begin{bmatrix} \cos\phi & 0 & \cos(\pi/2 - \phi) \\ 0 & 1 & 0 \\ \cos(\pi/2 + \phi & 0 & \cos\phi \end{bmatrix} \tag{4}$$

The geometry is now defined in two coordinate systems, and the means to transform from one system to the other is available. Equations 3 and 4 will be used frequently in the remainder of this report.

C. INITIAL CONDITIONS

The initial conditions consist of the initial positions of all points of interest and the initial velocity of the vehicle center of gravity and footpads. The initial vehicle geometry, which is the position vectors, is computed first. The symmetry of the vehicle allows a minimum amount of input data to be used in these calculations.

Four vehicle landing cases are considered, two for a vehicle with an even number of legs and two for a vehicle with an odd number of legs. One possible configuration for each of the four cases is shown in Figure 4.

Since all landing conditions are symmetric about the x'-axis, it is not necessary to compute the properties of all the legs. Let n be the number of legs on the vehicle and $\overline{m}$ the number of legs that must be considered; i.e., i will take values from 1 to $\overline{m}$. The relationships between $\overline{m}$ and n are

Case I $\qquad \overline{m} = n/2 \qquad$ (5)

Case II $\qquad \overline{m} = n/2 + 1 \qquad$ (6)

Cases III and IV $\qquad \overline{m} = (n + 1)/2 \qquad$ (7)

Now, the angle between the x'-axis and the number one leg is defined.

Cases I and IV $\qquad \Delta_3 = \pi/n \qquad$ (8)

Cases II and III $\qquad \Delta_3 = 0 \qquad$ (9)

The positions of interest in the body-fixed coordinate system are computed using the input data shown in Figure 5. The equations are

$$\Delta_1 = 2 \pi/n \tag{10}$$

$$\Delta_2 = \arcsin\left({}^{a}/R_2\right) \tag{11}$$

$$x'_{i1} = 0 \tag{12}$$

$$x'_{i2} = R_1 \cos\left[\Delta_3 + (i-1)\Delta_1\right] \tag{13}$$

$$x'_{i3} = R_2 \cos\left[\Delta_3 + (i-1)\Delta_1 + \Delta_2\right] \tag{14}$$

$$x'_{i4} = R_2 \cos\left[\Delta_3 + (i-1)\Delta_1 - \Delta_2\right] \tag{15}$$

$$x'_{i5} = R_3 \cos\left[\Delta_3 + (i-1)\Delta_1\right] \tag{16}$$

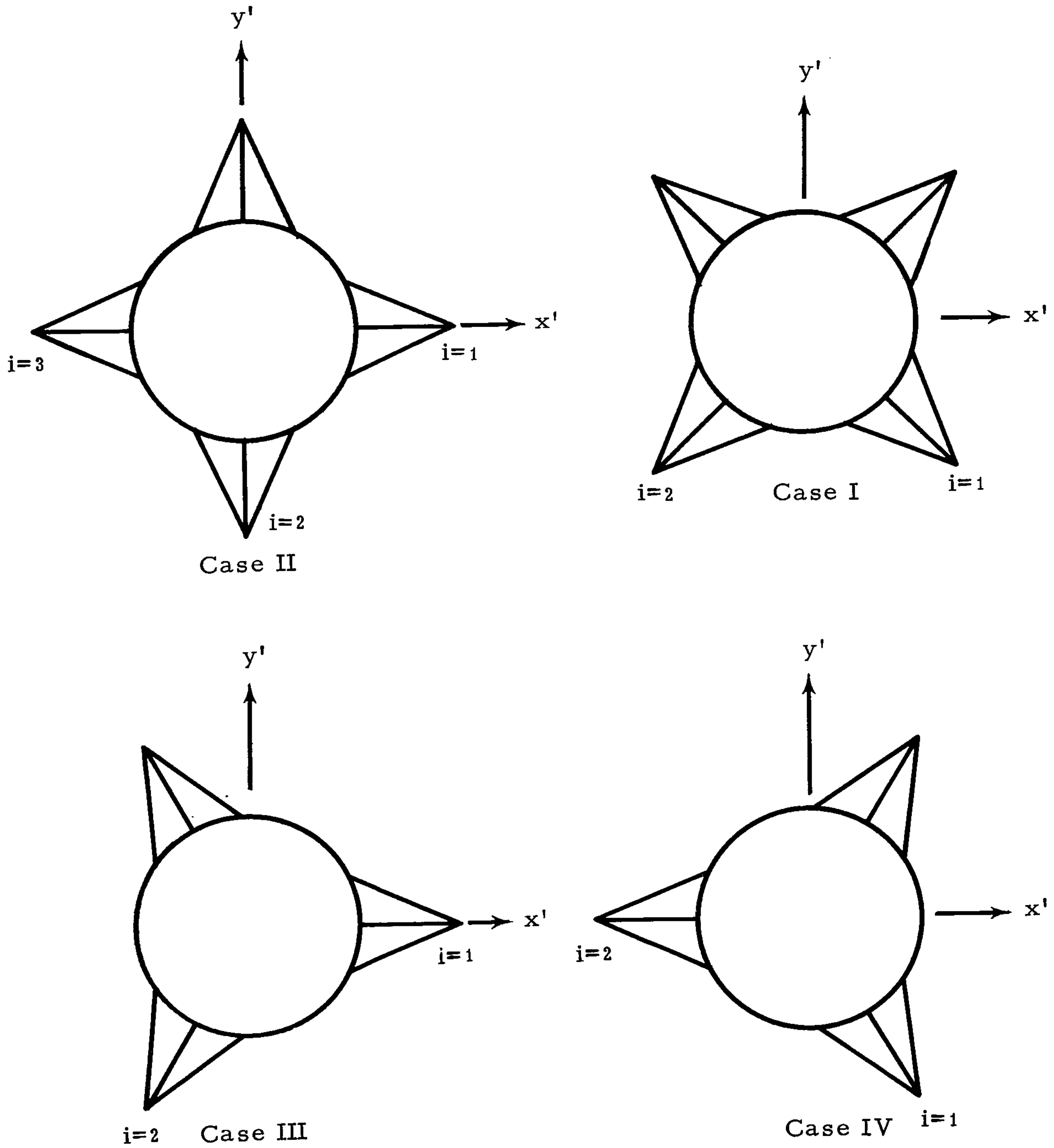

FIGURE 4. VEHICLE CONFIGURATION AND ORIENTATION OF THE FOUR LANDING CASES

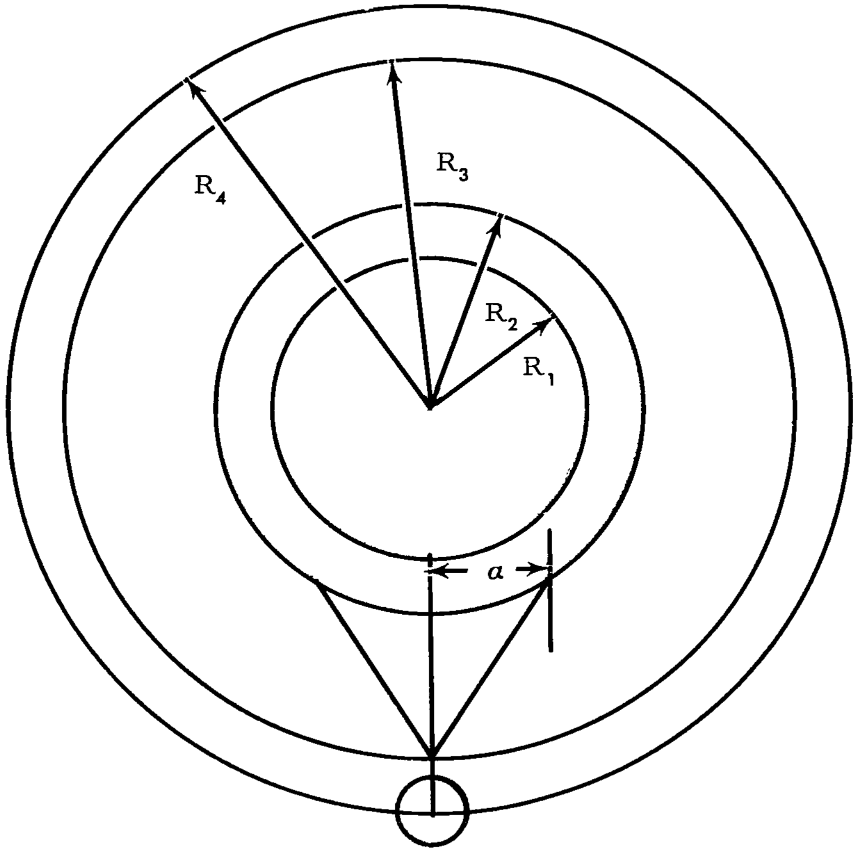

FIGURE 5. INITIAL VEHICLE CONFIGURATION GEOMETRY

$$x'_{i6} = R_4 \cos\left[\Delta_3 + (i-1)\Delta_1\right] \tag{17}$$

$$y'_{i1} = 0 \tag{18}$$

$$y'_{i2} = -R_1 \sin\left[\Delta_3 + (i-1)\Delta_1\right] \tag{19}$$

$$y'_{i3} = -R_2 \sin\left[\Delta_3 + (i-1)\Delta_1 + \Delta_2\right] \tag{20}$$

$$y'_{i4} = -R_2 \sin\left[\Delta_3 + (i-1)\Delta_1 - \Delta_2\right] \tag{21}$$

$$y'_{i5} = -R_3 \sin\left[\Delta_3 + (i-1)\Delta_1\right] \tag{22}$$

$$y'_{i6} = -R_4 \sin\left[\Delta_3 + (i-1)\Delta_1\right] \tag{23}$$

The values z_{1j}, j = 1 through 6 are also input data,

$$z'_{ij} = z'_{1j} \tag{24}$$

and

$$\phi = \beta + \theta \tag{25}$$

where β is the slope of the landing surface and θ is the angle between a vertical line and the longitudinal axis of the vehicle measured in the x-z plane.

After determining the position of all vehicle points in the body-fixed coordinate system, it is then necessary to define these points in the coordinate system fixed on the lunar surface. Initially, the point $P_{\overline{m}6}$ is set so $x_{\overline{m}6} = z_{\overline{m}6} = 0$. Using this condition the components of P_{i1} are found.

$$x_{i1} = \left[-x'_{\overline{m}6} - z_{\overline{m}1} \cos(\pi/2 - \phi)\right] / \cos\phi \tag{26}$$

$$y_{i1} = 0 \tag{27}$$

$$z_{i1} = -\left[z'_{\overline{m}6} \cos\phi - x'_{\overline{m}6} \cos(\pi/2 + \phi)\right] \tag{28}$$

The position vectors, in surface-fixed coordinates, of all the points of interest, can be computed next using equation 3. The initial lengths and angles are computed as follows:

$$\ell_{ijk} = \left[(x_{ij} - x_{ik})^2 + (y_{ij} - y_{ik})^2 + (z_{ij} - z_{ik})^2 \right]^{1/2} \tag{29}$$

where k = 5 and j = 2, 3, 4, 6.

$$\zeta_{i52,i56} = \cos^{-1} \left[\frac{(x_{i2} - x_{i5})(x_{i6} - x_{i5}) + (y_{i2} - y_{i5})(y_{i6} - y_{i5}) + (z_{i2} - z_{i5})(z_{i6} - z_{i5})}{\ell_{i25}\ \ell_{i65}} \right] \tag{30}$$

The initial velocity components of the center of gravity and the footpads are determined as follows:

$$\dot{\phi} = \dot{\theta} \tag{31}$$

$$\dot{x}_{i1} = (V_V \cos\theta + V_H \sin\theta) \sin\phi + (-V_V \sin\theta + V_H \cos\theta) \cos\phi \tag{32}$$

$$\dot{z}_{i1} = -(V_V \cos\theta + V_H \sin\theta) \cos\phi + (-V_V \sin\theta + V_H \cos\theta) \sin\phi \tag{33}$$

$$\dot{x}_{i6} = \dot{x}_{i1} + (z_{i1} - z_{i6})\ \dot{\theta} \tag{34}$$

$$\dot{y}_{i6} = 0 \tag{35}$$

$$\dot{z}_{i6} = \dot{z}_{i1} - (x_{i1} - x_{i6})\ \dot{\theta} \tag{36}$$

The remaining initial conditions are the parameters which describe the load stroke relationship of the deformable members.

These initial parameters are:

$$\Delta_{ij5} = (V_{ij5} + K_{\delta ij5}\ \delta_{ij5}) / K_{\delta ij5} \tag{37}$$

$$\Delta'_{ij5} = 0 \tag{38}$$

$$\overline{\Delta}_{ij5} = (-\overline{V}_{ij5} + K_{\dot{\delta} ij5}\ \overline{\delta}_{ij5}) / K_{\delta ij5} \tag{39}$$

$$\overline{\Delta}'_{ij5} = 0 \tag{40}$$

$$R_{ij5} = \ell_{ij5} \tag{41}$$

D. FORCES DUE TO DEFORMATION

During the time the vehicle is in contact with the lunar surface, forces will develop in its structure. These forces can be computed using the load-stroke characteristics of the deformable members. These members are assumed to be composed of a structural spring and a crushable material with no resilience. The structural spring is assumed to have linear characteristics which are the same in tension and compression (Fig. 6). The crushable material is assumed to have a linear onset rate to a maximum value and then remain constant for any additional deformation. The crushable material may exhibit different characteristics in tension and compression (Fig. 7).

The load-stroke characteristics of the structural springs and the crushable material may be combined in series, but it is important to realize that the combined load-stroke relationship changes as the member deforms. For the general case, the load-stroke curve is shown in Figure 8.

Combining the onset rate of the crushable material in series with the structural spring gives effective spring constants that are computed as follows:

$$K_{c_{ij5}} = K_{\delta ij5} \; V_{ij5} / (V_{ij5} + K_{\delta ij5} \; \delta_{ij5}) \tag{42}$$

$$\overline{K}_{c_{ij5}} = K_{\delta ij5} \; \overline{V}_{ij5} / (-\overline{V}_{ij5} + K_{\delta ij5} \; \overline{\delta}_{ij5}) \tag{43}$$

The general equation of the load-stroke characteristics of each member can now be written with the aid of a unit step function.

$$\begin{aligned}
F_{ij5} = {} & V_{ij5} \left[1 - U(\ell_{ij5} - R_{ij5} + \Delta_{ij5})\right] + \left[\Delta'_{ij5} K_{\delta ij5} + (R_{ij5} - \ell_{ij5} - \Delta'_{ij5}) \right. \\
& \left. K_{c_{ij5}}\right] \left[U(\ell_{ij5} - R_{ij5} + \Delta_{ij5}) - U(\ell_{ij5} - R_{ij5} + \Delta'_{ij5})\right] + \\
& \left[(R_{ij5} - \ell_{ij5}) \; K_{\delta ij5}\right] \left[U(\ell_{ij5} - R_{ij5} + \Delta'_{ij5}) - U(\ell_{ij5} - R_{ij5} - \Delta'_{ij5})\right] \\
& + \left[(-K_{\delta ij5} \overline{\Delta}'_{ij5}) + (R_{ij5} + \overline{\Delta}'_{ij5} - \ell_{ij5}) \; \overline{K}_{c_{ij5}}\right] \\
& \left[U(\ell_{ij5} - R_{ij5} - \overline{\Delta}'_{ij5}) - U(\ell_{ij5} - R_{ij5} - \overline{\Delta}_{ij5})\right] \\
& + \overline{V}_{ij5} \; U(\ell_{ij5} - R_{ij5} - \overline{\Delta}_{ij5})
\end{aligned} \tag{44}$$

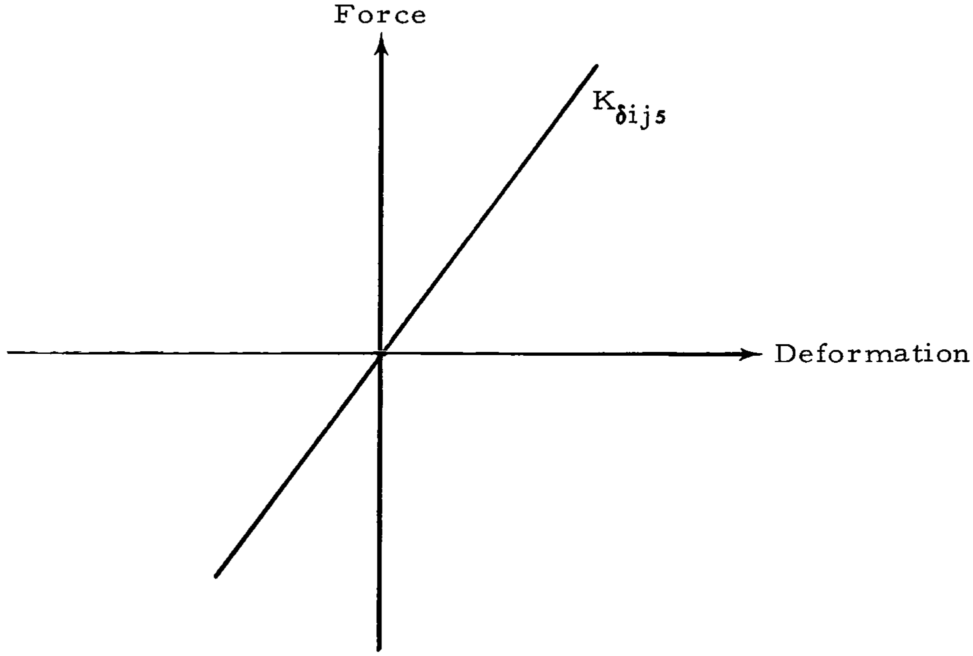

FIGURE 6. STRUCTURAL SPRINGS

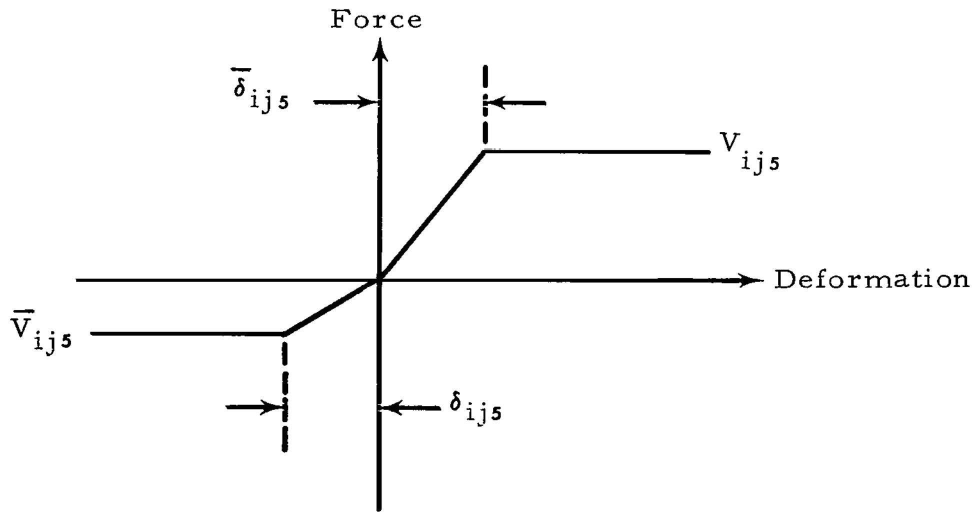

FIGURE 7. CRUSHABLE MATERIAL

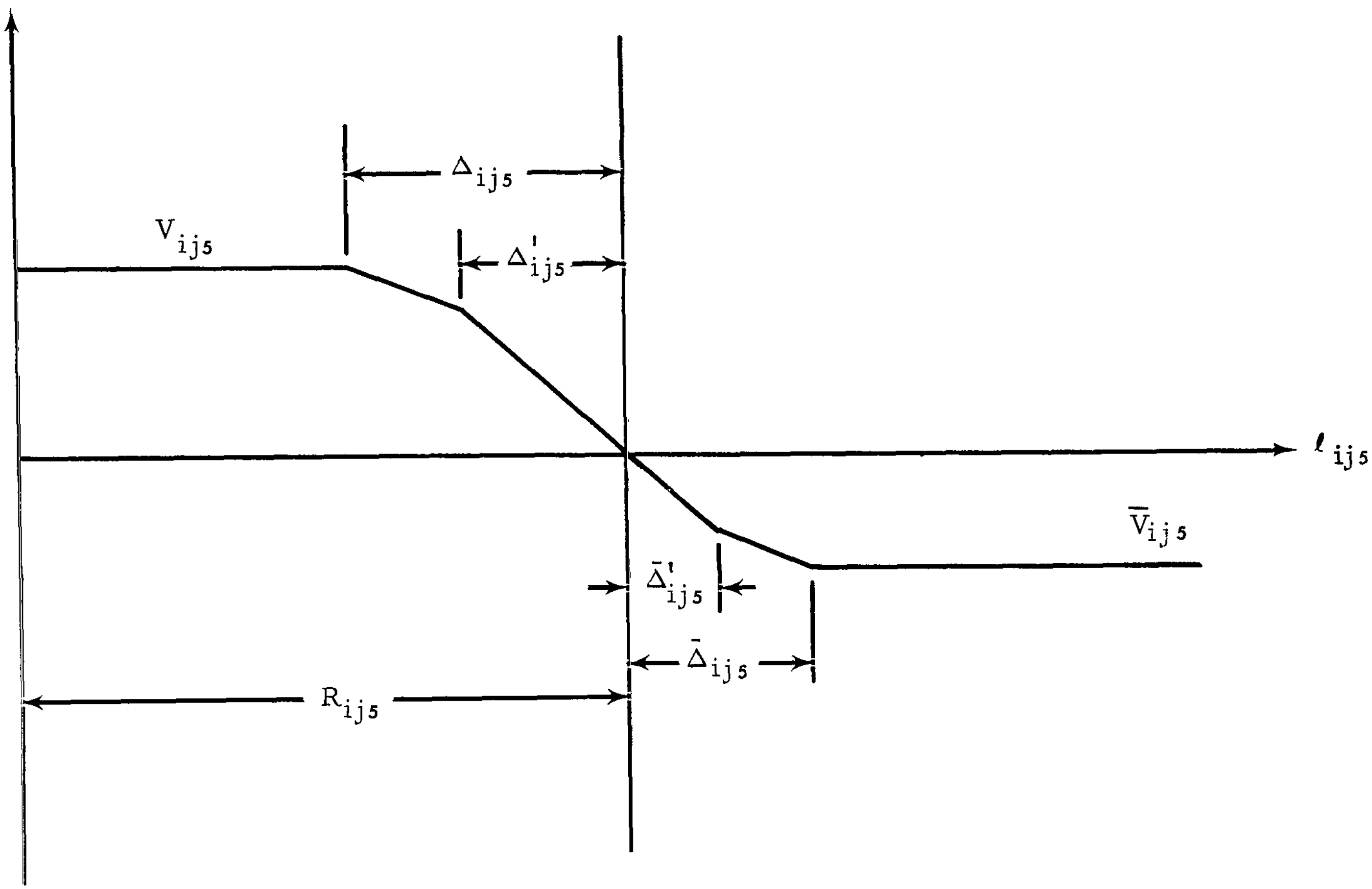

FIGURE 8. GENERAL LOAD STROKE CURVE

where U(x) is a unit step function,

$$U(x) = 0 \qquad x < 0$$

$$U(x) = 1 \qquad x \geq 0$$

Since the load-stroke relationship of a member is dependent on the history of the loads applied, it is necessary to compute the parameters which describe the shape of the load-stroke curve each time the member is loaded (i.e., for a given ℓ_{ij5} the force F_{ij5} is computed.). Then, R_{ij5}, Δ_{ij5}, Δ'_{ij5}, $\bar{\Delta}'_{ij5}$, and $\bar{\Delta}_{ij5}$ are changed so the load-stroke curve will be correct when another force is computed. The changes in the load-stroke curve are given as follows:

If $\qquad \ell_{ij5} < R_{ij5} - \Delta_{ij5}$

$$R^*_{ij5} = \ell_{ij5} + V_{ij5} / K_{\delta ij5}$$

$$\Delta^*_{ij5} = R^*_{ij5} - \ell_{ij5}$$

$$\Delta'^*_{ij5} = \Delta^*_{ij5}$$

$$\bar{\Delta}^*_{ij5} = \bar{\Delta}_{ij5}$$

$$\bar{\Delta}'^*_{ij5} = \bar{\Delta}'_{ij5} \tag{45}$$

If $\qquad R_{ij5} - \Delta_{ij5} < \ell_{ij5} < R_{ij5} - \Delta'_{ij5}$

$$R^*_{ij5} = \ell_{ij5} + \left[V_{ij5} - (\ell_{ij5} - R_{ij5} + \Delta_{ij5}) K_{c_{ij5}} \right] / K_{\delta ij5}$$

$$\Delta^*_{ij5} = \Delta_{ij5} - R_{ij5} + R^*_{ij5}$$

$$\Delta'^*_{ij5} = R^*_{ij5} - \ell_{ij5}$$

$$\bar{\Delta}^*_{ij5} = \bar{\Delta}_{ij5}$$

$$\bar{\Delta}'^*_{ij5} = \bar{\Delta}'_{ij5} \tag{46}$$

If $R_{ij_5} - \Delta'_{ij_5} < \ell_{ij_5} < R_{ij_5} + \bar{\Delta}'_{ij_5}$

$$R^*_{ij_5} = R_{ij_5}$$

$$\Delta^*_{ij_5} = \Delta_{ij_5}$$

$$\Delta'^*_{ij_5} = \Delta'_{ij_5}$$

$$\bar{\Delta}^*_{ij_5} = \bar{\Delta}_{ij_5}$$

$$\bar{\Delta}'^*_{ij_5} = \bar{\Delta}'_{ij_5} \tag{47}$$

If $R_{ij_5} + \bar{\Delta}'_{ij_5} < \ell_{ij_5} < R_{ij_5} + \bar{\Delta}_{ij_5}$

$$R^*_{ij_5} = \ell_{ij_5} + \left[\bar{V}_{ij_5} - (\ell_{ij_5} - R_{ij_5} - \bar{\Delta}_{ij_5}) \bar{K}_{c_{ij_5}}\right] / K_{\delta ij_5}$$

$$\Delta^*_{ij_5} = \Delta_{ij_5}$$

$$\Delta'^*_{ij_5} = \Delta'_{ij_5}$$

$$\bar{\Delta}^*_{ij_5} = R_{ij_5} + \bar{\Delta}_{ij_5} - R^*_{ij_5}$$

$$\bar{\Delta}'^*_{ij_5} = \ell_{ij_5} - R^*_{ij_5} \tag{48}$$

If $$R_{ij5} + \bar{\Delta}_{ij5} < \ell_{ij5}$$

$$R^*_{ij5} = \ell_{ij5} + \bar{V}_{ij5} / K_{\delta ij5}$$

$$\Delta^*_{ij5} = \Delta_{ij5}$$

$$\Delta'^*_{ij5} = \Delta'_{ij5}$$

$$\bar{\Delta}^*_{ij5} = \ell_{ij5} - R^*_{ij5}$$

$$\bar{\Delta}'^*_{ij5} = \bar{\Delta}^*_{ij5} \tag{49}$$

In equations 45 through 49, the values with an asterisk should be used in equation 44 to determine the force during the next time increment.

At this stage in calculation, all necessary equations have been developed to compute forces in the deformable members. These forces are now resolved into components and added to produce the force vectors acting parallel to the surface-fixed coordinates.

$$\bar{F}_{i6x} = \sum_{j=2}^{4} (x_{i5} - x_{ij}) \frac{F_{ij5}}{\ell_{ij5}} \tag{50}$$

$$\bar{F}_{i6y} = \sum_{j=2}^{4} (y_{i5} - y_{ij}) \frac{F_{ij5}}{\ell_{ij5}} \tag{51}$$

$$\bar{F}_{i6z} = \sum_{j=2}^{4} (z_{i5} - z_{ij}) \frac{F_{ij5}}{\ell_{ij5}} \tag{52}$$

This concludes the computation of the forces in the system. These forces will now be used in the equations of motion and must be determined for each integration time increment.

E. EQUATIONS OF MOTION

The equations of motion of the vehicle center of gravity and footpads will be used to determine its displacements, velocities, and accelerations during touchdown. These equations, for each vehicle configuration, are as follows:

Case I

$$\ddot{z}_{i1} = -\frac{2}{m} \sum_{i=1}^{\bar{m}} \bar{F}_{i6z} - g \cos \beta \tag{53}$$

$$\ddot{x}_{i1} = -\frac{2}{m} \sum_{i=1}^{\bar{m}} \bar{F}_{i6x} + g \sin \beta \tag{54}$$

$$\ddot{\phi} = -\frac{2}{I} \left\{ \sum_{i=1}^{\bar{m}} (x_{i6} - x_{i1}) \bar{F}_{i6z} + (z_{i1} - z_{i6}) \bar{F}_{i6x} \right\} \tag{55}$$

Case II

$$\ddot{z}_{i1} = -\frac{1}{m} \left\{ \bar{F}_{16z} + 2 \sum_{i=2}^{\bar{m}-1} \bar{F}_{i6z} + \bar{F}_{\bar{m}6z} \right\} - g \cos \beta \tag{56}$$

$$\ddot{x}_{i1} = -\frac{1}{m} \left\{ \bar{F}_{16x} + 2 \sum_{i=2}^{\bar{m}-1} \bar{F}_{i6x} + \bar{F}_{\bar{m}6x} \right\} + g \sin \beta \tag{57}$$

$$\ddot{\phi} = -\frac{1}{I} \Big\{ \bar{F}_{16z} (x_{16} - x_{11}) + 2 \sum_{i=2}^{\bar{m}-1} \bar{F}_{i6z} (x_{i6} - x_{i1}) + \bar{F}_{\bar{m}6z} (x_{\bar{m}6} - x_{m1})$$

$$+ F_{16x}(z_{11} - z_{16}) + 2 \sum_{i=2}^{\bar{m}-1} F_{i6x} (z_{i1} - z_{i6}) + \bar{F}_{\bar{m}6x}$$

$$(z_{\bar{m}1} - z_{\bar{m}6}) \Big\} \tag{58}$$

Case III

$$\ddot{z}_{i1} = -\frac{1}{\bar{m}} \left\{ \bar{F}_{16z} + 2 \sum_{i=2}^{\bar{\bar{m}}} \bar{F}_{i6z} \right\} - g \cos \beta \tag{59}$$

$$\ddot{x}_{i1} = -\frac{1}{m} \left\{ \bar{F}_{16x} + 2 \sum_{i=2}^{\bar{m}} \bar{F}_{i6x} \right\} + g \sin \beta \tag{60}$$

$$\ddot{\phi} = -\frac{1}{I} \left\{ \bar{F}_{16z} (x_{16} - x_{11}) + 2 \sum_{i=2}^{\bar{m}} \left[\bar{F}_{i6z} (x_{i6} - x_{i1}) + \bar{F}_{i6x} (z_{i1} - z_{i6}) \right] + \bar{F}_{16x} (z_{11} - z_{16}) \right\} \tag{61}$$

Case IV

$$\ddot{z}_{i1} = -\frac{1}{m} \left\{ 2 \sum_{i=1}^{\bar{m}-1} \bar{F}_{i6z} + \bar{F}_{\bar{m}6z} \right\} - g \cos \beta \tag{62}$$

$$\ddot{x}_{i1} = -\frac{1}{m} \left\{ 2 \sum_{i=1}^{\bar{m}-1} \bar{F}_{i6z} + \bar{F}_{\bar{m}6x} \right\} + g \sin \beta \tag{63}$$

$$\ddot{\phi} = -\frac{1}{I} \left\{ 2 \sum_{i=1}^{\bar{m}-1} \left[\bar{F}_{i6z} (x_{i6} - x_{i1}) + \bar{F}_{i6x} (z_{i6} - z_{i1}) \right] + \bar{F}_{\bar{m}6z} (x_{\bar{m}6} - x_{\bar{m}1}) + \bar{F}_{\bar{m}6x} (z_{\bar{m}6} - z_{\bar{m}1}) \right\} \tag{64}$$

The following equations are solved by assuming that the forces are constant for short periods of time:

$$Z_{i1} = Z^{*}_{i1} + \dot{Z}^{*}_{i1} \Delta t + \ddot{Z}_{i1} \frac{(\Delta t)^2}{2} \tag{65}$$

$$\dot{Z}_{i1} = \dot{Z}^{*}_{i1} + \ddot{Z}_{i1} \Delta t \tag{66}$$

$$X_{i1} = X^{*}_{i1} + \dot{X}^{*}_{i1} \Delta t + \ddot{X}_{i1} \frac{(\Delta t)^2}{2} \tag{67}$$

$$\dot{X}_{i1} = \dot{X}^{*}_{i1} + \ddot{X}_{i1} \Delta t \tag{68}$$

$$\phi = \phi^{*} + \dot{\phi}^{*} \Delta t + \ddot{\phi} \frac{(\Delta t)^2}{2} \tag{69}$$

$$\dot{\phi} = \dot{\phi}^{*} + \ddot{\phi} \Delta t \tag{70}$$

In these equations, the asterisk indicates the value of the particular parameter during the previous time increment.

The equations of motion of the footpads are handled in a somewhat different manner than are the equations of the center of gravity. When the footpads are of the lunar surface, they acted as part of a rigid body, and their motion is completely described by the motion of the center of gravity of the vehicle. When the footpads are on the lunar surface, they act as a mass with forces moving them. These applied forces are the friction forces and the forces from the deformable members of the vehicle as shown in Figure 9.

The equations of motion will now be developed for the case where the footpad is on the lunar surface. Summing the forces parallel to the x-axis we have

$$m_p \ddot{x}_{i6} + \overline{F}^{F}_{i6x} - \overline{F}_{i6x} = 0 \tag{71}$$

and paraliel to the y-axis

$$m_p \ddot{y}_{i6} + \overline{F}^{F}_{i6y} - \overline{F}_{i6y} = 0 \tag{72}$$

The total friction force acts along the line of the velocity vector of the footpad and is described as

$$\overline{F}^{F}_{i6x} = \frac{\mu \overline{F}^{N}_{i6z} \dot{x}_{i6}}{\sqrt{(\dot{x}_{i6})^2 + (\dot{y}_{i6})^2}} = - \frac{\mu \overline{F}_{i6z} \dot{x}_{i6}}{\sqrt{(\dot{x}_{i6})^2 + (\dot{y}_{i6})^2}} \tag{73}$$

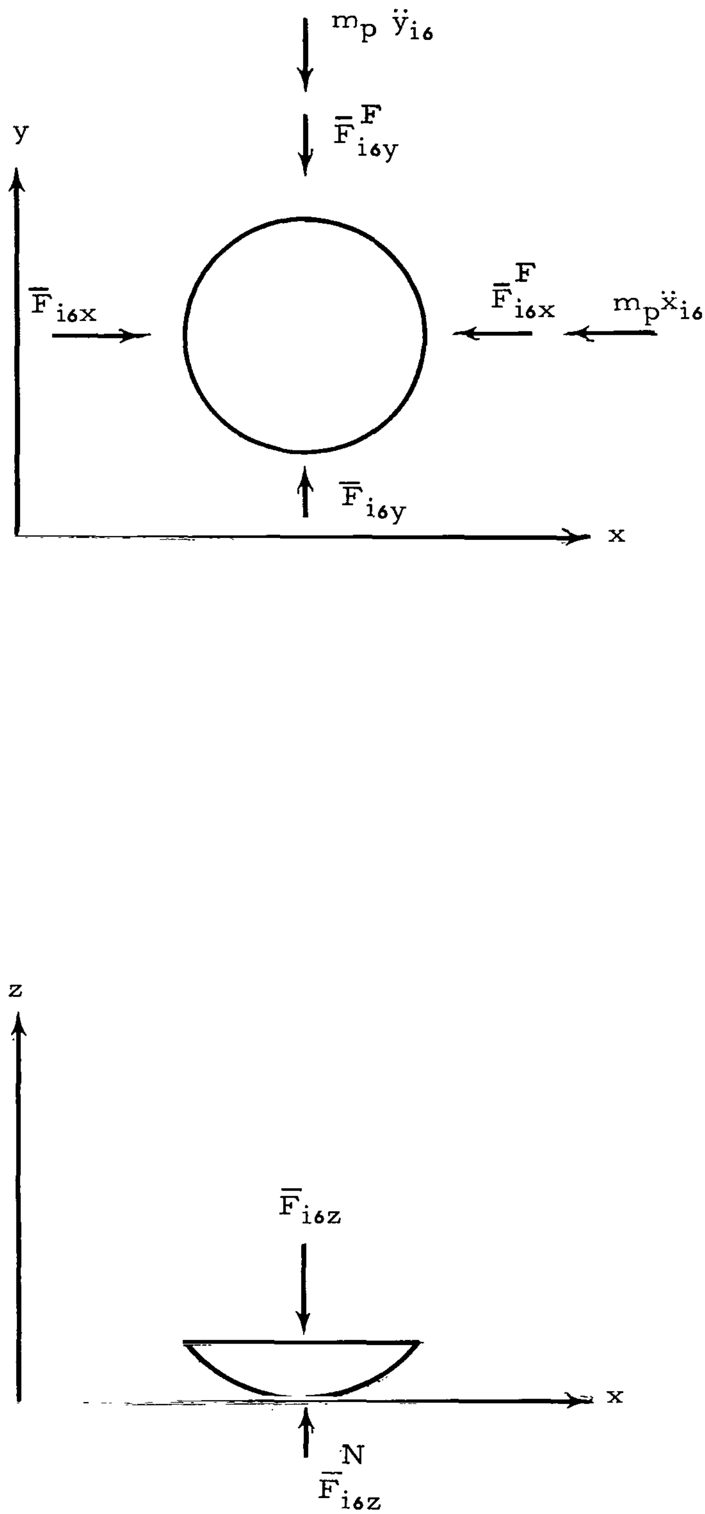

FIGURE 9. FORCES ACTING ON EACH FOOTPAD

$$\overline{F}^{F}_{i6y} = \frac{\mu \overline{F}^{N}_{i6z} \; \dot{y}_{i6}}{\sqrt{(\dot{x}_{i6})^2 \; (\dot{y}_{i6})^2}} = - \frac{\mu \overline{F}_{i6z} \; \dot{y}_{i}}{\sqrt{(\dot{x}_{i6})^2 \; (\dot{y}_{i6})^2}} \tag{74}$$

Now substituting equations 73 and 74 into the equations of motion, we obtain

$$\ddot{x}_{i6} - \frac{\mu \overline{F}_{i6z} \; \dot{x}_{i6}}{m_p \sqrt{(\dot{x}_{16})^2 + (\dot{y}_{16})^2}} - \frac{\overline{F}_{i6x}}{m_p} = 0 \tag{75}$$

$$\ddot{y}_{i6} - \frac{\mu \overline{F}_{i6z} \; \dot{y}_{i6}}{m_p \sqrt{(\dot{x}_{i6})^2 + (\dot{y}_{i6})^2}} - \frac{\overline{F}_{i6y}}{m_p} = 0 \tag{76}$$

If the assumption is made that the forces and resultant velocities are constant for each time increment of integration, then the equations of motion are simple, linear differential equations with the solution

$$x_{i6} = x_{i6}^{*} + D\,(C + \dot{x}_{i6}^{*}) \; (1 - e^{-\frac{\Delta t}{D}}) - C\Delta t \tag{77}$$

$$\dot{x}_{i6} = (C + \dot{x}_{i6}^{*}) \; e^{-\Delta t/D} - C \tag{78}$$

$$y_{i6} = \dot{y}_{i6}^{*} + D\,(E + \dot{y}_{i6}^{*}) \; (1 - e^{-\frac{\Delta t}{D}}) - E\Delta t \tag{79}$$

$$y_{i6} = (E + \dot{y}_{i6}^{*}) e^{-\frac{\Delta t}{D}} - E \tag{80}$$

where

$$C = \frac{\overline{F}_{i6x} \, (\dot{x}_{i6}^{*2} + \dot{y}_{i6}^{*2})^{1/2}}{\mu \overline{F}_{i6z}}$$

$$D = - \frac{m_i \; (\dot{x}_{i6}^{*2} + \dot{y}_{i6}^{*2})^{1/2}}{\mu \overline{F}_{i6z}}$$

$$E = \frac{\overline{F}_{i6y} \; (\dot{x}_{i6}^{*2} + \dot{y}_{i6}^{*2})^{1/2}}{\mu \overline{F}_{i6z}}$$

and the values with the asterisk are for the previous time point.

Equations 77 through 80 describe the motion of the footpads if they are on the surface during the complete time increment. If they are off the surface during the complete time increment, the position vectors and velocities are

$$P_{i6} = P_{i1} + A\, P'_{i6} \tag{81}$$

where P'_{i6} does not change since the leg is acting as part of a rigid body, and

$$\dot{x}_{i6} = \dot{x}_{i1} + (z_{i1} - z_{i6})\dot{\phi} \tag{82}$$

$$\dot{y}_{i6} = 0 \tag{83}$$

$$\dot{z}_{i6} = \dot{z}_{i1} - (x_{i1} - x_{i6})\dot{\phi} \tag{84}$$

There is also the case where the footpad is off the surface at the start of the time increment but was on the surface at the end of the time increment. This case is handled by assuming that z_{i6} changes much more than x_{i6} or y_{i6} during the time increment. Based on this assumption, we compute P_{i6} using equation 81 and set $z_{i6} = 0$.

With the coordinates of the center of gravity of the vehicle and footpads being known, the position vectors of all other points on the vehicle can be computed. If the footpads do not touch the surface, these position vectors can be computed using equation 3 since all values of P'_{ij} are constant. While the footpads are on the surface, the members deform, and a more involved method of computing the positions is used. This method is a follows:

$$P_{i2} = P_{i1} + A\, P'_{i2} \tag{85}$$

$$P_{i3} = P_{i1} + A\, P'_{i3} \tag{86}$$

$$P_{i4} = P_{i1} + A\, P'_{i4} \tag{87}$$

$$P'_{i6} = B\,(P_{i6} - P_{i1}) \tag{88}$$

The only position vectors not known are P_{i5} and P'_{i5} is found by

$$\ell_{i62} = \left[(x_{i6} - x_{i2})^2 + (y_{i6} - y_{i2})^2 + (z_{i6} - z_{i2})^2\right]^{1/2} \tag{89}$$

$$A_{i62} = \left[(x'_{i6} - x'_{i2})^2 + (y'_{i6} - y'_{i2})^2\right]^{1/2} \tag{90}$$

$$\gamma_{i62} = \cos^{-1}\left(\frac{A_{i62}}{\ell_{i62}}\right) \tag{91}$$

$$\gamma_{i65} = \gamma_{i62} + \pi - \zeta_{i52,\, i56} - \sin^{-1}\left[\frac{\ell_{i65}}{\ell_{i62}} \sin(\zeta_{i52,\, i56})\right] \tag{92}$$

$$A_{i65} = \ell_{i65} \cos \gamma_{i65} \tag{93}$$

$$x'_{i5} = x'_{i6} - A_{i65}\left(\frac{x'_{i6} - x'_{i2}}{A_{i62}}\right) \tag{94}$$

$$y'_{i5} = y'_{i6} - A_{i65}\left(\frac{y'_{i6} - y'_{i2}}{A_{i62}}\right) \tag{95}$$

$$z'_{i5} = z'_{i6} + \ell_{i65} \sin \gamma_{i65} \tag{96}$$

$$P_{i_5} = P_{i1} + A \quad P'_{i5} \tag{97}$$

These equations complete the positioning of all points of interest on the vehicle in both body-fixed and surface-fixed coordinate systems. The length of the deformable members can now be computed by

$$\ell_{ij5} = \left[(x_{ij} - x_{i5})^2 + (y_{ij} - y_{i5})^2 + (z_{ij} - z_{i5})^2\right]^{1/2} \tag{98}$$

$$j = 2, 3, 4$$

This completes the computation necessary for each time increment. Now the complete time history is obtained with the procedure shown in Figure 1.

SECTION III. STABILITY CRITERIA

Stability of the vehicle is determined by comparing the positions of the center of gravity of the vehicle and its footpads. The horizontal position of these points must be known to make these comparisons. These positions can be computed as follows:

$$x_{H11} = x_{11} \cos \beta + z_{11} \cos (\pi/2 - \beta) \tag{99}$$

$$x_{H16} = x_{16} \cos \beta + z_{16} \cos (\pi/2 - \beta) \tag{100}$$

$$x_{H\overline{m}6} = x_{\overline{m}6} \cos \beta + z_{\overline{m}6} \cos (\pi/2 - \beta) \tag{101}$$

The conditions for instability are

$$x_{H11} > x_{H16} \text{ and } x_{H11} < x_{H\overline{m}6} \tag{102}$$

The stability of the vehicle is computed at each time increment, and when instability occurs, the time history is stopped. If instability does not occur, the time history is stopped after maximum loads develop and the motion of the vehicle has subsided to a point where stability is ensured.

SECTION IV. REMARKS ON COMPUTATION

This method of analyzing the touchdown dynamics of a lunar vehicle requires the use of a large, high-speed computer. Because of the large number of computations, numerical problems may arise and require the use of double precision in the computation. Any computer other than a high-speed machine would not be practical because of the large amount of machine time required for each landing study.

The computer used for programing this analysis was an IBM 7094. A double-precision Fortran program prepared by Mr. John Southan of the Computation Laboratory, MSFC, was used to obtain the solution of this analysis.

NASA-Langley, 1965

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